

# Whither the Anyon?\*

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It is pointed out that Chern-Simons field theories do not always allow an interpretation in terms of anyons even in the nonrelativistic limit. Significantly, when the two approaches are found to differ the former is the one which is compatible with established physical principles. Thus the role left for the anyon is that of a very special subcase of Chern-Simons theories in the limit in which relativistic and spin considerations become irrelevant.

The anyon [1] is a concept which has been widely heralded in recent years as a potentially useful tool in superconductivity theory. That this idea should gain such wide acceptance is perhaps not too surprising since the possibility that physics could include more general types of statistics than bosonic or fermionic ones has considerable appeal. Nonetheless, it must clearly be recognized that the successes of anyonism are really only variants of a single case – namely, the interaction of two (or more) spinless nonrelativistic particles through an Aharonov-Bohm [2] potential. Such a system is described in the CM frame by the two dimensional differential equation

$$\left(\frac{1}{i}\nabla + \alpha\epsilon_\phi\frac{1}{r}\right)^2\psi = k^2\psi, \quad (1)$$

where  $\alpha$  is a flux parameter and  $\epsilon_\phi$  a unit vector in the  $\phi$  direction. Since

$$\left(\epsilon_\phi\frac{1}{r}\right)_i = -\epsilon_{ij}\nabla_j\ln r,$$

one can, if one uses the, (almost correct) relation

$$\epsilon_{ij}\nabla_j\ln r = -\nabla_i\tan y/x, \quad (2)$$

reduce (1) to

$$-\nabla^2\psi' = k^2\psi',$$

where

$$\psi' = e^{-i\alpha\phi}\psi.$$

In other words, the interaction is eliminated and the wave function becomes nonperiodic (i.e., anyonic).

However, this result cannot be generalized to include either spin effects or special relativity.

The failure to accommodate special relativity stems from the fact [3] that (2) is not a valid equation (a gradient generally cannot equal the dual of a gradient). This means that the correction of (2) prevents one from doing successfully in quantum field theory what one can achieve in nonrelativistic quantum mechanics. This is essentially because the former involves extended sources while the latter deals only with point particles.

Difficulties with spin are seen to occur when one reduces the Chern-Simons field theory for a spin-1/2 field [4] to the nonrelativistic limit. One finds in this case that (1) becomes

$$k^2\psi_\pm = \left[\left(\frac{1}{i}\nabla \pm \frac{\epsilon\lambda^2}{2\pi}\epsilon_\phi\frac{1}{r}\right)^2 + \frac{\epsilon\lambda^2}{2\pi}\frac{1}{r}\delta(r)\right]\psi_\pm, \quad (3)$$

where the  $\pm$  notation refers to the spin up/down cases,  $\epsilon = \pm 1$ , and  $\lambda$  is a coupling constant. The delta function acts as a repulsive potential for  $\epsilon = 1$  and an attractive one when  $\epsilon = -1$ . For  $\epsilon = 1$  the delta function (i.e., the Zeeman term) keeps the particles away from the origin and the anyon construction can be carried out. However for  $\epsilon = -1$  one finds that solutions which are singular at  $r = 0$  become relevant [5] and the anyon approach fails. This is because the Zeeman interaction induces a transition from a regular to an irregular solution.

This can also be seen by trying to mimic for the Dirac equation

$$E\psi = \left[\beta\gamma \cdot \left(\frac{1}{i}\nabla + \alpha\epsilon_\phi\frac{1}{r}\right) + \beta m\right]\psi$$

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the elimination of the interaction term which was accomplished for (1). One finds that (2) would allow this to be accomplished and thus lead to a free Dirac equation. When this is converted to a second order differential equation, the Zeeman term is absent! Thus incorrect physics is predicted in which the spin variable plays no role in the interaction. (Recall also that helicity is conserved in a time independent magnetic field, thus requiring a precession of the spin in the scattering process.)

A striking illustration of the inadmissibility of the anyon interpretation when  $\varepsilon = -1$  is provided by a calculation of the second virial coefficient for a system of spin up or spin down particles. It is straightforward using techniques of [6] to obtain for the spin down case the virial coefficient  $B_2(\alpha_-, T)$  at temperature  $T$ . In terms of the thermal wavelength  $\lambda_T = (2\pi M k T)^{1/2}$ ,

$$B_2(\alpha_-, T) = \frac{1}{4} \lambda_T^2 \begin{cases} 1 - 2\beta^2 & N \text{ even} \\ 1 - 2(\beta - 1)^2 & N \text{ odd, } N < 0 \\ 1 - 2(\beta + 1)^2 & N \text{ odd, } N > 0, \end{cases}$$

where  $\alpha_- = N + \beta$  with  $N$  an integer and  $0 \leq \beta < 1$ . The result is plotted in Figure 1. It is sufficient to note that the discontinuities in  $B_2$  and the nonperiodicity in  $\alpha$  are both at variance with the anyon interpretation.

In summary, the anyon is an idea which applies only to the case of nonrelativistic quantum mechanics when spin can safely be ignored. It is merely a very special case of Chern-Simons field theory.

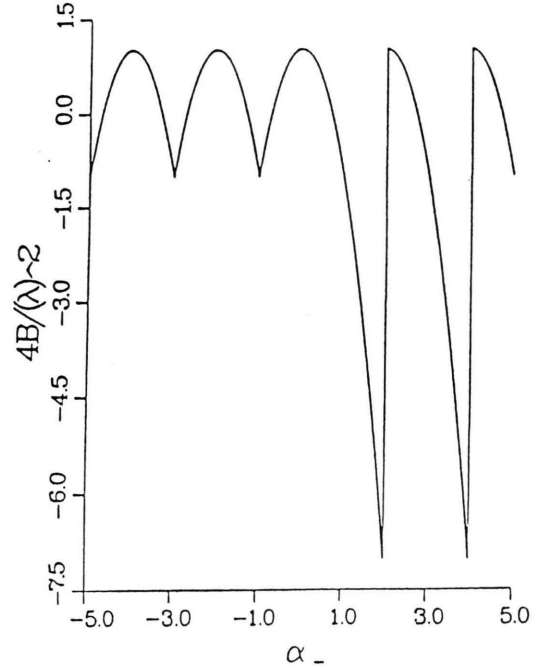


Fig. 1. The virial coefficient  $B_2(\alpha_-, T)$  for the spin down case.

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